

The Silent Mean: Hidden Decimal Recursion, Perfect Fifth Correspondence, and the Symmetry of the Dual-Track Shimmer

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Abstract

The dual-track shimmer of Precise Temperament was originally introduced as a two-frequency system generated by the exact shimmer fold

$$S = \frac{250047}{250000} = 1.000188.$$

While investigating the decimal architecture of the resulting frequencies, a previously unnoticed numerical phenomenon emerged.

For many perfect-fifth relationships, the decimal extension of the destination Track 2 frequency appears recursively within the arithmetic mean between Track 1 and Track 2.

This paper documents the observed decimal correspondences, distinguishes direct matches from octave-wrapped recursions, and proposes that the arithmetic mean functions as the hidden axis of symmetry of the shimmer.

The paper concludes with a symbolic interpretation relating this mathematical structure to the triadic geometry traditionally represented by Shiva's Trident.

1 Introduction

The dual-track shimmer was introduced as a means of relocating the structural comma of temperament into a uniform temporal modulation.

Each note possesses two frequencies,

$$T_2 = T_1 S,$$

where

$$S = \frac{250047}{250000}.$$

The shimmer therefore consists only of Track 1 and Track 2.

Initially the arithmetic mean

$$M = \frac{T_1 + T_2}{2}$$

was regarded as merely a numerical convenience.

Subsequent investigation suggests otherwise.

The arithmetic mean appears repeatedly within the decimal architecture itself and occupies a unique mathematical role in the symmetry of the shimmer.

2 The Decimal Extension Phenomenon

Consider the reference note

$$A = 432.081216 \text{ Hz.}$$

Its pure perfect fifth is

$$432.081216 \times \frac{3}{2} = 648.121824.$$

Remarkably,

$$648.121824$$

is not Track 1 of E.

It is Track 2.

The corresponding three values become

$$648.000000,$$

$$648.060912,$$

$$648.121824.$$

The decimal extensions form a perfectly symmetric progression,

$$0.000000 \rightarrow 0.060912 \rightarrow 0.121824,$$

with the arithmetic mean containing exactly one-half of the total shimmer displacement.

The same structure appears repeatedly throughout the scale.

Examples include

$$486.000000 \rightarrow 486.045684 \rightarrow 486.091368,$$

$$594.000000 \rightarrow 594.055836 \rightarrow 594.111672,$$

and

$$792.000000 \rightarrow 792.074448 \rightarrow 792.148896.$$

The decimal architecture therefore appears recursively partitioned before arriving at the final shimmer frequency.

3 Matching and Wrapped Correspondence

Two distinct behaviours are observed.

3.1 Direct Matching

For many notes, the perfect fifth lands directly upon another Track 2 frequency.

The arithmetic mean then reveals the exact midpoint of the decimal displacement.

These correspondences suggest that the shimmer fold is not merely an offset but participates in the recursive fifth structure itself.

3.2 Octave Wrapping

Other notes do not terminate directly upon a listed Track 2 frequency.

Rather than representing failures, these appear to wrap through the octave lattice.

The destination frequency exists mathematically but lies outside the displayed twenty-four-position table.

The recursion therefore continues while the tabulated representation terminates.

This distinction explains why some decimal correspondences appear explicit while others remain hidden.

4 The Hidden Mean

Although only Track 1 and Track 2 are sounded, the resulting waveform is

$$\cos(2\pi f_1 t) + \cos(2\pi f_2 t).$$

Applying the standard trigonometric identity gives

$$2 \cos(\pi(f_2 - f_1)t) \cos\left(2\pi \frac{f_1 + f_2}{2} t\right).$$

The oscillation therefore possesses a carrier centred exactly upon

$$\frac{f_1 + f_2}{2},$$

the arithmetic mean.

No third oscillator is required.

The arithmetic mean emerges naturally from the interaction of the two played frequencies.

It is therefore mathematically present despite never being independently generated.

5 A Hidden Triadic Structure

The dual-track shimmer appears, at first glance, to consist of only two frequencies.

Closer examination suggests that every shimmer possesses three mathematical components:

1. Track 1
2. The Arithmetic Mean
3. Track 2

Only the outer two are sounded.

The centre exists implicitly as the axis of symmetry.

Rather than representing an additional note, the arithmetic mean represents the equilibrium point about which the shimmer is organised.

6 Symbolic Interpretation

The preceding observations are entirely mathematical.

One possible symbolic interpretation is suggested by the traditional image of Shiva's Trident (Trisūla).

The two outer frequencies correspond to the two outer prongs.

The arithmetic mean corresponds to the silent central prong.

Unlike the outer frequencies, the central element is never independently struck.

Nevertheless, it silently governs the symmetry of the entire structure.

The shimmer therefore arises not merely from duality but from an underlying triadic organisation whose centre is implicit rather than explicit.

This symbolic analogy should not be regarded as a mathematical proof.

Rather, it provides an evocative conceptual framework through which the hidden symmetry may be understood.

7 Discussion

The decimal extension phenomenon appears to reveal a recursive numerical architecture embedded within the shimmer itself.

Three observations stand out.

1. Many perfect fifths terminate directly upon Track 2 frequencies.
2. The arithmetic mean consistently partitions the decimal displacement into two equal halves.
3. Frequencies apparently lacking direct correspondence are plausibly explained by octave wrapping beyond the displayed lattice.

Whether these decimal relationships represent a deeper invariant of the recursive generation process remains an open question deserving further investigation.

8 Conclusion

The arithmetic mean was originally introduced as a numerical midpoint.

Its repeated appearance within the decimal architecture, its emergence as the carrier of the dual-track waveform, and its correspondence with many perfect-fifth shimmer destinations suggest that it occupies a far more fundamental role.

Rather than being an additional frequency requiring performance, it may represent the hidden axis about which the shimmer itself is organised.

The shimmer is therefore not merely a duality.

It appears to possess an implicit third principle:

a silent centre that is never directly heard, yet whose presence is mathematically unavoidable.

“The visible frequencies sing.
The invisible centre holds them in perfect balance.”

References

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